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NUMERICAL SIMULATION OF VORTEX BREAKDOWN

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16. Abstract

In the present work, the breakdown of an isolated axisymmetric vortex embedded in an unbounded uniform flow is examined by numerical integration of the complete Navier-Stokes equations for unsteady axisymmetric flow. The results show that if the vortex strength is small, the solution approaches a steady flow and the vortex is stable and that if this strength is large enough, the solution remains unsteady and a recirculating zone will appear near the axis, its form and internal structure resembling those of the axisymmetric breakdown bubbles with multi-cells observed by Faler and Leibovich (1978). For appropriate combinations of flow parameters, the flow reveals quasi-periodicity. Parallel calculations with the quasi-cylindrical approximation indicate that so far as predicting of breakdown is concerned, its results coincide quite well with the results mentioned above. They both show that the vortex breakdown has little concern with the Reynolds number or with the critical classification of the upstream flow, at least for the lower range of Reynolds numbers covered by the calculations of this work.

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Numerical Simulation of Vortex Breakdown

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I. Introduction

Since Peckham and Atkinson (1957) first observed the breakdown of the leading edge vortex of a large sweptback wing at large angles of attack, many authors introduced various theoretical explanations, such as the critical flow theory by Benjamin (1962, 1967), the unstable theory of fluid dynamics by Ludwieg (1962) and the quasi-cylindrical approximation theory by Gartshore (1962, 1963), Hall (1965, 1966, 1967) and Mager (1972),** for the breakdown of vortices. Until now, however, not a single theory is widely accepted. There is considerable confusion among various theories, as well as between theoretical and experimental values. */22

Based on either the critical flow theory or the finite transition theory, the flow upstream from the breakdown must be super critical. Hall (1967, 1972) and Ludwieg (1970) also pointed out that the vortex breakdown process described by quasi-cylindrical approximation is just the process in which a super critical flow approaches a critical state. This seems to be a popular viewpoint. That means the breakdown of a vortex must start from a super critical upstream flow. Lavan, Nielsen, and Fejer (1969), Kopecky and Torrance (1973), and Grabowski and Berger (1976) directly performed numerical integration of the

*Numbers in margin indicate pagination in foreign text

**And Squire (1960), Bossel (1967, 1969).

complete Navier-Stokes equations regardless of whether the upstream flow is supercritical or subcritical, resulting in an axisymmetric breakdown bubble recirculating zone (see Figure 1) similar to that observed in the experiment which is directly contradictory to the above theory. This contradiction is yet to be clarified.

On the other hand, their numerical results could not show the double cell structure inside the breakdown bubble (see Figure 1) as measured by a laser flow meter by Faler and Leibovich (1978).

Furthermore, when the rotational speed is high enough, their /23
computation could not even get a convergent solution. This may be due to their excessively rigorous assumption. Leibovich (1978) believed that if the periodicity and asymmetry of the flow inside the breakdown bubble observed experimentally were not considered, then any numerical experiment could not describe this double cell structure.

In this work, attention is paid to their opinion. Through axisymmetry and numerical integration of the complete unsteady N-S equations, we hope to more realistically describe the breakdown of "axisymmetric" vortices. The axisymmetric assumption remains because of economic considerations and limitation of computer capability. In order to facilitate the analysis and comparison, parallel calculations are made under quasi-cylindrical

approximation. On this basis, the contradiction between the theoretical results and numerical experiments is discussed.

II. Mathematical Model

In this work, an isolated axisymmetric vortex with a constant circulation embedded in a uniform flow in an infinite space is investigated. Let us take a cylindrical coordinate and make the x-axis coincide with the axis of symmetry. Let us assume that the axial and circumferential velocity distributions on a certain "inlet cross-section" are expressed by the two following equations:

$$x = 0; \quad \begin{cases} u = 1 + \alpha f(r) \\ w = \Gamma_0 g(r) \end{cases}$$

where the velocity components and coördinates are rendered dimensionless by the uniform flow velocity at infinity u_∞ and the vortex core radius R. Figure 2 schematically shows their shapes. The initial axial velocity or the axis $U_0 = 1+\alpha$ and the velocity circulation, or vortex intensity Γ_0 at infinity are two flow parameters.

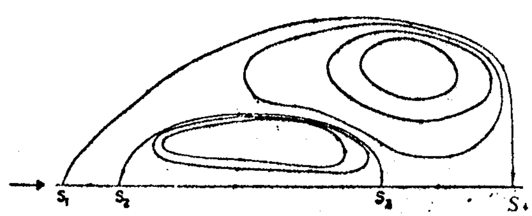


Figure 1. The Flow Field Structure Inside an Asymmetric Breakdown Bubble (Leibovich 1978).

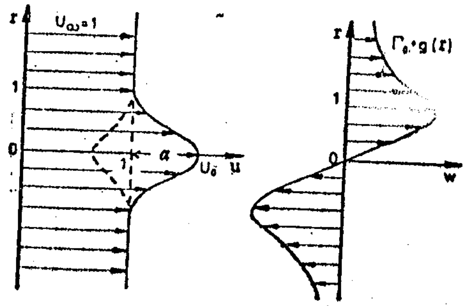


Figure 2. Initial Axial and Tangential Velocity Distributions

If we introduce the local circulation $\Gamma = rw$

the circumferential vortex component $\Omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial r}$

and the flow function Ψ

then the dimensionless N-S equations for the unsteady, axisymmetric motion of a viscous, incompressible fluid may be re-written a series of Γ-Ω-Ψ equations:

/24

$$
\begin{cases}
\frac{\partial \Gamma}{\partial t} + u\frac{\partial \Gamma}{\partial x} + v\frac{\partial \Gamma}{\partial r} = \frac{1}{Re}\left[\frac{\partial^2 \Gamma}{\partial x^2} + r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Gamma}{\partial r}\right)\right] \\
\frac{\partial \Omega}{\partial t} + u\frac{\partial \Omega}{\partial x} + v\frac{\partial \Omega}{\partial r} - \frac{v\Omega}{r} - \frac{\partial}{\partial x}\frac{\Gamma^2}{r^3} = \frac{1}{Re}\left\{\frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (r\Omega)}{\partial r}\right]\right\} \\
\frac{\partial^2 \Psi}{\partial x^2} + r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Psi}{\partial r}\right) = r\Omega \\
u = 1 - \frac{1}{r}\frac{\partial \Psi}{\partial r}, \qquad v = \frac{1}{r}\frac{\partial \Psi}{\partial x}
\end{cases}
$$

where $Re = u_\infty R/\nu$. This is a series of parabolic equations. In terms of spatial coordinates, however, they are also elliptical.

In order to obtain a real solution, it is necessary to provide the appropriate initial conditions as well as all the boundary conditions over the region of integration.

The region of integration is defined as follows:

$$D=\{0\leq x\leq L,\ 0<r<+\infty\}$$

The exit boundary is chosen to be sufficiently downstream at x=L, where L>>1, such as L=20.

The condition for a definite solution is specified as follows:

initial condition:

t=0: $\Gamma=\Gamma_0 rg(r)$, $\Omega= -\alpha f'(r)$, $\Psi= -\alpha\int_0^r rf(r)dr$ in D

boundary condition:

$$
\begin{array}{llll}
x=0; & \Gamma=\Gamma_0 r\,g(r), & \Omega=-\alpha f'(r), & \Psi=-\alpha\int_0^r r\,f(r)dr; \\
x=L; & \dfrac{\partial\Gamma}{\partial x}=0, & \dfrac{\partial\Omega}{\partial x}=0, & \dfrac{\partial^2\Psi}{\partial x^2}=0; \\
r=0; & \Gamma=0, & \Omega=0\ , & \Psi=0; \\
r\to+\infty; & \Gamma\to\Gamma_0, & \Omega\to 0\ , & \dfrac{\partial\Psi}{\partial r}\to 0.
\end{array}
$$

The so-called quasi- cylindrical approximation is to assume that

$$v<<u,w \qquad \frac{\partial}{\partial x}<<\frac{\partial}{\partial r}$$

and the flow is steady. Under this approximation, the above series of equations can be simplified as:

$$
\begin{cases}
u\dfrac{\partial \Gamma}{\partial x}+v\dfrac{\partial \Gamma}{\partial r}=\dfrac{r}{Re}\dfrac{\partial}{\partial r}\left(\dfrac{1}{r}\dfrac{\partial \Gamma}{\partial r}\right)\\
u\dfrac{\partial \Omega}{\partial x}+v\dfrac{\partial \Omega}{\partial r}-\dfrac{v\Omega}{r}-\dfrac{\partial}{\partial x}\dfrac{\Gamma^2}{r^3}=\dfrac{1}{Re}\dfrac{\partial}{\partial r}\left[\dfrac{1}{r}\dfrac{\partial (r\Omega)}{\partial r}\right]\\
u=1+\displaystyle\int_r^{\infty}\Omega dr\\
\Psi=\displaystyle\int_0^r r(1-u)dr\\
v=\dfrac{1}{r}\dfrac{\partial \Psi}{\partial x}
\end{cases}
$$

This is also a series of parabolic equations. When the initial conditions on the initial cross-section x=0 and the boundary /25 conditions at the axis r=0 and the external edge $r\rightarrow +\infty$ are given, numerical solution can always be obtained by iteration along the x-direction. The initial conditions given are:

$$x=0: \quad \Gamma=\Gamma_0 rg(r), \quad \Omega=-a f'(r) \qquad (A)$$

Then, the boundary conditions are

$$
\begin{aligned}
&r=0: && \Gamma=0 && \Omega=0\\
&r\rightarrow+\infty: && \Gamma\rightarrow\Gamma_0, && \Omega\rightarrow 0
\end{aligned}
\qquad (B)
$$

Near the breakdown point of the vortex, however, the quasi-cylindrical approximation is no longer valid. The differential equations also become unstable. Numerical calculation is no longer convergent. Thus, the presence of a large axial gradient in the calculation or the divergence of the computation can be as a label for a vortex breakdown.

In order to turn an infinite integration zone in the r-direction into a finite one and to ensure that the numerical solution

has a high enough resolution in the region where the flow changes vigorously, two independent coordinate transformations are introduced radially and axially:

$$x = c(e^{b\xi} - 1) \qquad x: (0, L) \to \xi: (0, 1)$$

$$r = \tan\eta \qquad r: (0, +\infty) \to \eta: \left(0, \frac{\pi}{2}\right)$$

In quasi-cylindrical approximation, only the radial transformation is required. All differential equations, as well as the initial and boundary conditions, must be transformed accordingly.

III. Results and Discussion

The Crank-Nicolson mean implicit finite difference method is used to solve the simplified equations under quasi-cylindrical approximation. The method is simple and efficient. The result shows that when Γ_0 is very small, such as $\Gamma_0 = 0.63$, the axial velocity varies slightly along the axis. If we proceed to calculate downstream, the axial velocity slows down initially and then gradually rises to approach the incoming flow velocity $U \to 1$. The vortex is stable. As Γ_0 increases, the axial velocity drops faster and faster. When Γ_0 exceeds a certain value, the axial velocity abruptly drops to a value close to 0 at a certain position. The computation is no longer convergent from this point. As discussed earlier, this position can be considered as the vortex breakdown point. With increasing Γ_0, the breakdown point position continues moving upstream. Increasing U_0, however, will strengthen the stability of the vortex. The legends shown in Figure 3 represent a combination of results

obtained by using quasi-cylindrical approximation. From the figure we can see that, in terms of vortex breakdown, the conclusion obtained with Re=100 is in total agreement with Re=200 with the exception of one point $U_0=1.4$ and $r_0=0.8944$. The dotted lines in the figure represent the boundary of vortex breakdown.

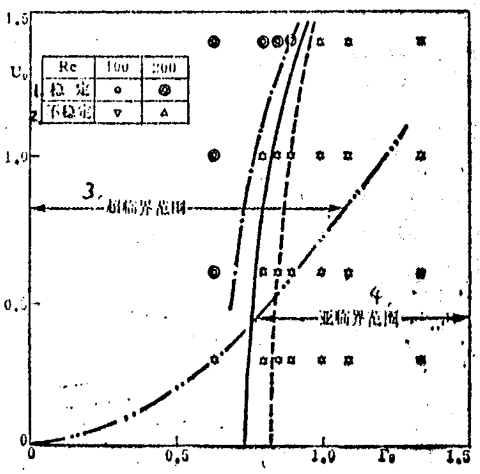


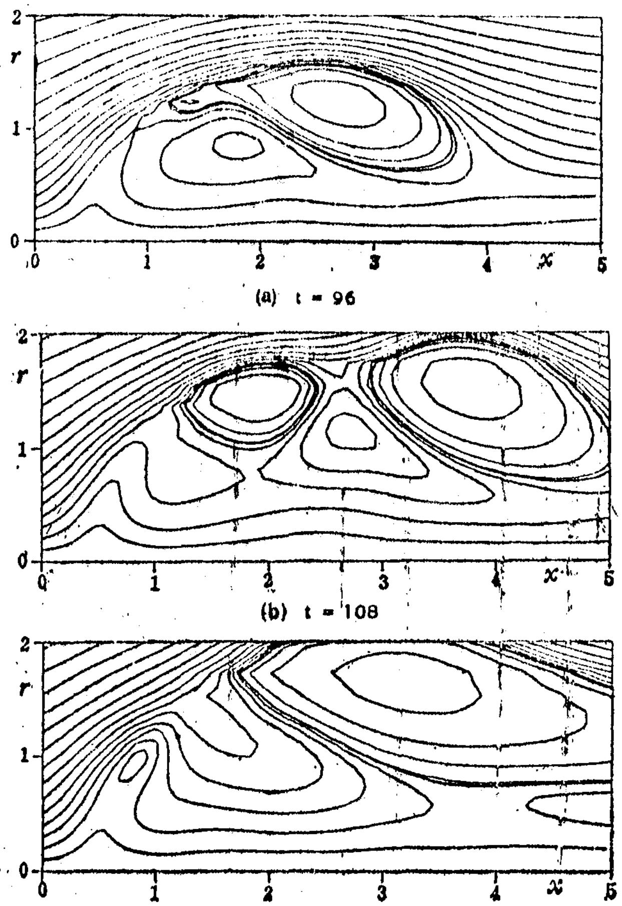
Figure 3. Calculated Results

1. stable
2. unstable
3. super critical range
4. subcritical range

The complete N-S equations are solved by alternate direction /26 iterations. The result shows that any stable vortex flow as predicted by the quasi-cylindrical approximation will reach a steady state after some time. The flow surface appears to be flat. The quasi-cylindrical approximation should obviously be valid in the entire flow field. A comparison of velocity distribution also shows that the flow fields obtained by both methods agree extremely well. This indicates that quasi-cylindrical approximation is indeed an excellent approximation

for stable vortices. Furthermore, the numerical integration program designed for the complete N-S equations was also tested.

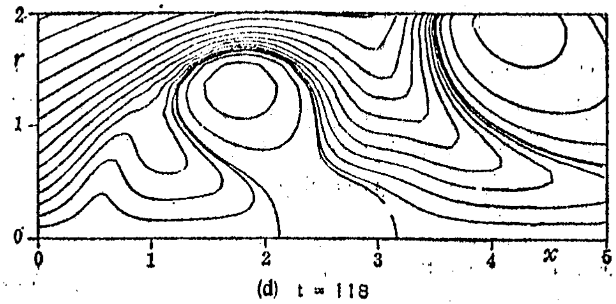
When vortex breakdown is predicted by quasi-cylindrical approximation, with the exception of individual edges, the numerical solution of the complete N-S equations shows that the flow is unsteady. Figure 4 is one of the examples. They are intercepts of an axisymmetric flow plane and a meridian plane. With increasing time, it begins to bulge near the axial flow plane and then develops into an enclosed recirculating zone. This recirculating zone continues to develop into a so-called "double cell" structure (see Figure 4a). Its appearance and internal structure is very similar to the broken cell (Figure 1) measured by Faler and Leibovich (1978) using a laser flow meter. With an appropriate combination of flow parameters, the flow appears to be quasi-periodic after some time. A new internal cell is formed periodically at the head of a broken cell. It gradually strengthens and moves along with the main stream. Then, it either combines with the inner cell formed earlier to become a large cell and flow downstream, or flows away alone. In Figure 4, b-d and e-g are very similar in sequence, which can be interpreted by the periodicity of the solution. Qualitatively, the periodicity of the unstable vortex



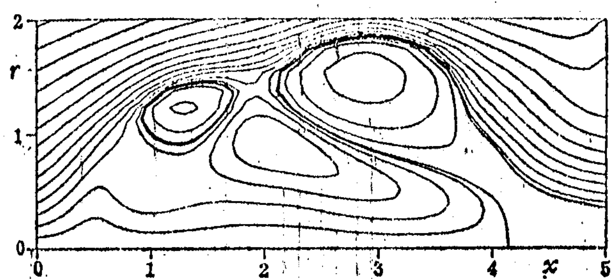
(a) t = 96

(b) t = 108

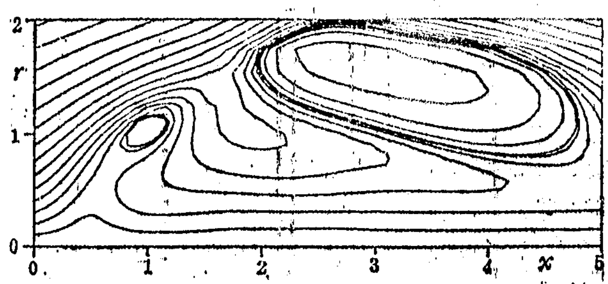
(c) t = 114



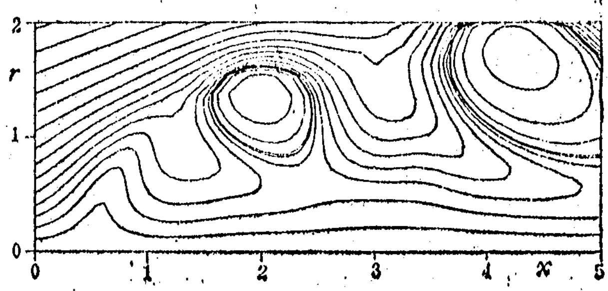
(d) t = 118 /27



(e) t = 122



(f) t = 126



(g) t = 130

motion discovered in our numerical calculation is in agreement with the experimental observations made by Sarpkaya (1971), and Faler and Leibovich (1978).

If we assume that the flow becomes unstable with respect to /28 any non-axisymmetric perturbation after an axisymmetric cell breakdown occurs, then the periodic inner cells flowing downstream may be the spiral tail behind a broken cell as

observed in experiments.

When the complete N-S equations are used in the calculation, the conclusions obtained are in total agreement with those using the quasi-cylindrical approximation. In terms of whether a vortex breaks down, the results are identical with Re=100 and 200. The solid line in Figure 3 represents the vortex breakdown boundary calculated based on the complete unsteady N-S equations. It stands between the curve obtained based on quasi-cylindrical approximation and the curve (dotted line) obtained in using the complete steady N-S equations by Grabowski and Berger (1976). They are, however, very close to one another.

Figure 3 also plots the critical curve (double dot dotted line) separating the upstream supercritical and subcritical regions as calculated by Mager (1972) based on the equation introduced by Benjamin (1962). From the figure we can see that many breakdown solutions are obtained with subcritical upstream conditions; just as Grabowski and Berger (1976) pointed out earlier. It does not agree with the critical flow theory by Squire and the finite transition theory by Benjamin. A parallel calculation based on quasi-cylindrical approximation also proved that these subcritical upstream flows will lead to vortex breakdown. The parabolic nature of the equations under quasi-cylindrical approximation precludes the possibility of any perturbation propagating upstream. Therefore, the breakdown of these vortices thus calculated cannot be explained by the propagation of disturbance upstream. Various experiments

conducted in pipes also show that when the flow rate remains unchanged and the rotation is intensified, which means when the subcritical nature of the upstream flow is strengthened based on Mager's critical curve, what happens is the upward shift of the breakdown point and the change of the pattern, rather than the disappearance of the breakdown of the vortex is, is indeed independent of the critical nature of the upstream flow.

IV. Conclusions

In this work, the breakdown of an isolated axisymmetric vortex embedded in a uniform flow is investigated by the numerical integration of the complete N-S equations under the low Reynolds numbers.

First, the N-S equations are simplified using the quasi-cylindrical approximation. Its solution can be determined by proceeding along the x-direction by a numerical method. The results show that this method is an excellent approximation of the real flow for stable vortices. The rapid drop of axial velocity and the divergence of the calculation can be considered as a sign of vortex breakdown. In addition, the breakdown of a vortex is very sensitive to the variation of vortex intensity.

The numerical integration of the complete N-S equations for an unsteady axisymmetric flow, however, shows that the solution approaches a steady state if vortex breakdown does not occur. Otherwise, the solution will remain unsteady. A recirculating zone will appear near the axial line. Its shape and internal

structure is very similar to the broken cell observed experimentally by Faler and Leibovich (1978). With appropriate combination of flow parameters, the flow will appear to be quasi-periodic after some time.

The consistency of both methods indicates that the breakdown of a vortex does not concern the Reynolds number significantly, at least in the lower Reynolds number range calculated. Furthermore, it is not related to the classification of the critical status of the upstream motion.

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